

Optimized Strength Model of Cement-RHA-Laterite-Water Mix in Hollow Sandcrete Block

P.N. Onuamah

Civil Engineering Department, Enugu State University of Science and Technology, Enugu, Nigeria.

Abstract-With the cost of cement and transportation of sandcrete aggregates in Nigeria increasing geometrically, designers now seek more locally available materials for constructing durable structures that still stand the test of time. This paper focuses on modeling and optimizing the compressive strength of a four component mixture of Cement, Rice Husk Ash, Laterite and Water in the production of hollow sandcrete block, at 67/33 Cement/RHA and 100% laterization. The study applies the Scheffe's optimization approach to obtain a mathematical model of the form $f(x_{11}, x_{12}, x_{13}, x_{14})$, where x_i are proportions of the concrete components, viz: cement, RHA, laterite and water. Scheffe's experimental design techniques are followed to mould various hollow block samples measuring 450mm x 225mm x 150mm and tested for 28 days strength. The task involved experimentation and design, applying the second order polynomial characterization process of the simplex lattice method. The model adequacy is checked using the control factors. A purpose-made software is prepared to handle the design computation process to take the desired property of the mix, and generate the optimal mix ratios.

Index terms: Laterization, Sandcrete, Pseudo-component, Simplex-lattice, Modeling, structural concrete, laterization.

INTRODUCTION

Concrete is the main material of civil engineering construction, and the ease or cost of its production water accounts for the level of success in environmental upgrading viz.: construction of roads, buildings, dams, structures and the renovation of such structures. To produce the concrete several primary components such as cement, sand, gravel and some admixtures are to be present in varying quantities and qualities. Unfortunately, the occurrence and availability of these components vary very randomly with location and hence the attendant problems of either excessive or scarce quantities of the different materials occurring in different areas. Where the scarcity of one component prevails exceedingly, the cost of the concrete production increases geometrically. Such problems obviate the need to seek alternative materials for partial or full replacement of the scarce component when it is possible to do so without losing the quality of the concrete.

1.1 Optimization Concept

Every activity that must be successful in human endeavour requires planning. The target of planning is the maximization of the desired outcome of the venture. In order to maximize gains or outputs it is often necessary to keep inputs or

investments at a minimum at the production level.

The process involved in this planning activity of minimization and maximization is referred to as optimization [1]. In the science of optimization, the desired property or quantity to be optimized is referred to as the objective function. The raw materials or quantities whose amount of combinations will produce this objective function are referred to as variables.

The variations of these variables produce different combinations and have different outputs. Often the space of variability of the variables is not universal as some conditions limit them. These conditions are called constraints. For example, money is a factor of production and is known to be limited in supply. Hence, an optimization process is one that seeks for the maximum or minimum value and at the same time satisfying a number of other imposed requirements [2].

Structural concrete is made with specified materials for specified strength. Concrete is heterogeneous as it comprises sub-materials. Concrete is made up of fine aggregates, coarse aggregates, cement, water, and sometimes admixtures. [3] report that modern research in concrete seeks to provide greater understanding of its constituent materials and possibilities of improving its qualities. For instance, Portland cement has been partially replaced with ground

granulated blast furnace slag (GGBS), a by-product of the steel industry that has valuable cementitious properties [4]. The task of concrete mix optimization implies selecting the most suitable concrete aggregates from the data base [5]. Several methods have been applied. Examples are by [6], [7], and [8]. [9] proposed an approach which adopts the equilibrium mineral assemblage concept of geochemical thermodynamics as a basis for establishing mix proportions. [10] reports that optimization of mix designs require detailed knowledge of concrete properties. Low water-cement ratios lead to increased strength but will negatively lead to an accelerated and higher shrinkage.

1.2 Modeling

Modeling means setting up mathematical equations of physical or other systems. Many factors of different effects occur simultaneously dependently or independently in different materials. The interplaying effects inter-affect one another differently at equal, direct, combined or partially combined rates to generate varied natural constants in the form of coefficients and/or exponents. The challenging problem is to understand and assess these distinctive constants by which the interplaying factors underscore some unique natural phenomenon towards which their natures tend, in a single, double or multi phase systems.

For such assessment a model could be constructed for a proper observation of response from the interaction of the factors through controlled experimentation followed by schematic design as in the simplex lattice approach of the type of Henry Scheffe [11] optimization theory. Also entirely different physical systems may correspond to the same mathematical model so that they can be solved by the same methods. This is an impressive demonstration of the unifying power of mathematics [12].

2. Literature Review

A good structural material should be homogeneous and isotropic. The Portland cement, laterite or concrete are none of these, nevertheless they are popular construction materials [13] Laterized concrete can be used in constructing cylindrical storage structures [14]. With given

proportions of aggregates the compressive strength of concrete depends primarily upon age, cement content, and the cement-water ratio [15]. Tropical weathering (laterization) is a prolonged process of chemical weathering which produces a wide variety in the thickness, grade, chemistry and ore mineralogy of the resulting soils [16].

The mineralogical and chemical compositions of laterites are dependent on their parent rocks [16] Laterite formation is favoured in low topographical reliefs of gentle crests and plateaus which prevent the erosion of the surface cover [17].

Of all the desirable properties of hardened concrete such as the tensile, compressive, flexural, bond, shear strengths, etc., the compressive strength is the most convenient to measure and is used as the criterion for the overall quality of the hardened concrete [2].

According Oluremi [18], hollow sandcrete blocks produced with RHA are widely acceptable to minimize the cost of construction works as the cost of sandcrete production is drastically reduced. Also a survey by the Raw Material Research and Development Council of Nigeria reveals that certain building materials deserve serious consideration as substitute for imported ones. Few of these include cement/lime stabilized blocks, adobe soil blocks, clay blocks, rice husk ash, lime and stonecrete blocks. Rice Husk, when burnt under controlled conditions, is highly pozzolanic and very suitable for use in lime-pozzolana mixes and for Portland cement replacement [19].

Mathematical models have been used to optimize some mechanical properties of concrete made from Rice Husk Ash (RHA), - a pozzolanic waste [1] and [20]. [21] show that the inclusion of mound soil in mortar matrix resulted in a compressive strength value of up to 40.08N/mm², and the addition of 5% of mound soil to a concrete mix of 1:2:4:0.56 (cement: sand: coarse aggregate: water) resulted in an increase of up to 20.35% in compressive strength.

Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture, and they are equidistant from each other [22].

When studying the properties of a q-component mixture, which are dependent on the component ratio only the factor space is a regular (q-1)-simplex [23]. Simplex lattice designs are saturated, that is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 ($x_i = 0, 1/m, 2/m, \dots, 1$), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used [23].

3. Scheffe's Theory

This is a theory where a polynomial expression of any degrees, is used to characterize a simplex lattice mixture components. In the theory only a single phase mixture is covered. The theory lends path to a unifying equation model capable of taking varying components ratios to fix approximately equal mixture properties. The optimization is the selectability, from some criterial (mainly economic) view point, the optimal ratio from the component ratios list that can be automatically generated. Scheffe's theory is one of the adaptations to this work in the formulation of response function for compressive strength of hollow sandcrete block.

3.1 Simplex Lattice

Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture [22], and they are equidistant from each other. Mathematically, a simplex lattice is a space of constituent variables of $X_1, X_2, X_3, \dots, X_i$ which obey these laws:

$$\left. \begin{array}{l} X_i \neq \text{negative} \\ 0 \leq x_i \leq 1 \\ \sum_{i=1} x_i = 1 \end{array} \right\} \dots\dots\dots (1)$$

That is, a lattice is an abstract space. To achieve the desired strength of concrete, one of the essential factors lies on the adequate proportioning of ingredients needed to make the concrete. [11] developed a model whereby if a mixture property desired is specified, possible combinations of needed ingredients to achieve the

compressive strength can easily be predicted by the aid of computer, and if proportions are specified the compressive strength can easily be predicted.

3.2 Simplex Lattice Method

In designing experiment to attack mixture problems involving component property diagrams the property studied is assumed to be a continuous function of certain arguments and with a sufficient accuracy it can be approximated with a polynomial [23]. When investigating multi-components systems the use of experimental design methodologies substantially reduces the volume of an experimental effort. Further, this obviates the need for a spatial representation of complex surface, as the wanted properties can be derived from equations while the possibility to graphically interpret the result is retained.

As a rule the response surfaces in multi-component systems are very intricate. To describe such surfaces adequately, high degree polynomials are required, and hence a great many experimental trials. A polynomial of degree n in q variable has C^{n+q} coefficients. If a mixture has a total of q components and X_i be the proportion of the i^{th} component in the mixture such that,

$$X_i \geq 0 \quad (i=1,2, \dots,q), \quad \dots \dots \dots (2)$$

then, the sum of the component proportion is a whole unity i.e.

$$X_1 + X_2 + X_3 + X_4 = 1 \text{ or } \sum X_i - 1 = 0 \dots \dots (3)$$

where $i = 1, 2, \dots, q$... Thus the factor space is a regular (q-1) dimensional simplex. In (q-1) dimensional simplex if $q = 2$, we have 2 points of connectivity. This gives a straight line simplex lattice. If $q=3$, we have a triangular simplex lattice and for $q = 4$, it is a tetrahedron simplex lattice, etc. Taking a whole factor space in the design we have a (q,m) simplex lattice whose properties are defined as follows:

- i. The factor space has uniformly distributed points,
 - ii. Simplex lattice designs are saturated [23].
- That is,

the proportions used for each factor have $m + 1$ equally spaced levels from 0 to 1 ($x_i = 0, 1/m, 2/m, \dots, 1$), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used.

Hence, for the quadratic lattice (q,2), approximating the response surface with the second degree polynomials (m=2), the following levels of every factor must be used 0, 1/2 and 1; for the tetrahedric (m=4) polynomials, the levels are 0, 1/4, 2/4, 3/4 and 1, etc; [11] showed that the number of points in a (q,m) lattice is given by

$$C_{q+m-1} = q(q+1) \dots (q+m-1)/m! \dots \quad (4)$$

3.2.1 (4,2) Lattice Model

The properties studied in the assumed polynomial are real-valued functions on the simplex and are termed responses. The mixture properties were described using polynomials assuming a polynomial function of degree m in the q-variable X_1, X_2, \dots, X_q , subject to Eqn (1), and will be called a (q,m) polynomial having a general form:

$$\hat{Y} = b_0 + \sum_{i=1}^q b_i X_i + \sum_{i < j} b_{ij} X_i X_j + \dots + \sum_{i < j < k} b_{ijk} + \sum_{i_1 < i_2 < \dots < i_n} b_{i_1 i_2 \dots i_n} X_{i_1} X_{i_2} \dots X_{i_n} \quad (5)$$

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{24} X_2 X_4 + b_{23} X_2 X_3 + b_{34} X_3 X_4 + b_{11} X_1^2 + b_{22} X_2^2 + b_{33} X_3^2 + b_{44} X_4^2 \dots \quad (6)$$

where b is a constant coefficient.

The relationship obtainable from Eqn (6) is subjected to the normalization condition of Eqn. (3) for a sum of independent variables. For a ternary mixture, the reduced second degree polynomial can be obtained as follows:

From Eqn. (3)

$$X_1 + X_2 + X_3 + X_4 = 1 \quad (7)$$

i.e

$$b_0 X_2 + b_0 X_2 + b_0 X_3 + b_0 X_4 = b_0 \dots$$

(8)

Multiplying Eqn. (3.7) by X_1, X_2, X_3, X_4 , in succession gives

$$X_1^2 = X_1 - X_1 X_2 - X_1 X_3 - X_1 X_4$$

$$X_2^2 = X_2 - X_1 X_2 - X_2 X_3 - X_2 X_4 \dots \dots$$

(9)

$$X_3^2 = X_3 - X_1 X_3 - X_2 X_3 - X_3 X_4$$

$$X_4^2 = X_4 - X_1 X_4 - X_2 X_4 - X_3 X_4$$

Substituting Eqn (8) into Eqn (9), we obtain after necessary transformation that

$$\hat{Y} = (b_0 + b_1 + b_{11}) X_1 + (b_0 + b_2 + b_{22}) X_2 + (b_0 + b_3 + b_{33}) X_3 + (b_0 + b_4 + b_{44}) X_4 + (b_{12} - b_{11} - b_{22}) X_1 X_2 + (b_{13} - b_{11} - b_{33}) X_1 X_3 + (b_{14} - b_{11} - b_{44}) X_1 X_4 + (b_{23} - b_{22} - b_{33}) X_2 X_3 + (b_{24} - b_{22} - b_{44}) X_2 X_4 + (b_{34} - b_{33} - b_{44}) X_3 X_4 \dots \dots \dots (10)$$

If we denote

$$\beta_i = b_0 + b_i + b_{ii}$$

and $\beta_{ij} = b_{ij} - b_{ii} - b_{jj}$,

then we arrive at the reduced second degree polynomial in 6 variables:

$$\hat{Y} = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4 \quad (11)$$

Thus, the number of coefficients has reduced from 15 in Eqn (6) to 10 in Eqn (11). That is, the reduced second degree polynomial in q variables is

$$\hat{Y} = \sum \beta_i X_i + \sum \beta_{ij} X_i X_j \dots \dots \quad (12)$$

3.2.2 Construction of Experimental/Design Matrix

From the coordinates of points in the simplex lattice, we can obtain the design matrix. We recall that vertex A the principal coordinates of the lattice, only the A-component is 1 (refer to fig 1), others are zero.

Table 1 Design matrix for (4,2) Lattice

N	X_1	X_2	X_3	X_4	Y_{exp}
1	1	0	0	0	Y_1

2	0	1	0	0	Y ₂
3	0	0	1	0	Y ₃
4	0	0	0	1	Y ₄
5	1/2	1/2	0	0	Y ₁₂
6	1/2	0	1/2	0	Y ₁₃
7	1/2	0	0	1/2	Y ₁₄
8	0	1/2	1/2	0	Y ₂₃
9	0	1/2	0	1/2	Y ₂₄
10	0	0	1/2	1/2	Y ₃₄

Hence if we substitute in Eqn (11), the coordinates of the first point (X₁=1, X₂=0, and X₃=0, Table 1), we get that Y₁= β₁.

And doing so in succession for the other two points if the tetrahedron, we obtain

$$Y_2 = \beta_2, Y_3 = \beta_3, \text{ and } Y_4 = \beta_4. \quad \dots \quad (13)$$

The substitution of the coordinates of the fourth point yields

$$Y_{12} = \frac{1}{2} X_1 + \frac{1}{2} X_2 + \frac{1}{2} X_1 \cdot \frac{1}{2} X_2 \\ = \frac{1}{2} \beta_1 + \frac{1}{2} \beta_2 + \frac{1}{4} \beta_{12}$$

But as β_i = Y_i then

$$Y_{12} = \frac{1}{2} \beta_1 + \frac{1}{2} \beta_2 + \frac{1}{4} \beta_{12}$$

Thus

$$\beta_{12} = 4 Y_{12} - 2Y_1 - 2Y_2 \quad \dots \quad (14)$$

And similarly,

$$\beta_{13} = 4 Y_{13} - 2Y_1 - 2Y_3$$

$$\beta_{23} = 4 Y_{23} - 2Y_2 - 2Y_3$$

Or generalizing,

$$\beta_i = Y_i \text{ and } \beta_{ij} = 4 Y_{ij} - 2Y_i - 2Y_j \quad \dots (15)$$

which are the coefficients of the reduced second degree polynomial for a q-component mixture, since the three points defining the coefficients β_{ij} lie on the edge. The subscripts of the mixture property symbols indicate the relative content of each component X₁ alone and the property of the mixture is denoted by Y₁. Mixture 4 includes X₁ and X₂, and the property being designated Y₁₂.

3.2.3 Actual and Pseudo Components

The requirements of the simplex that

$$\sum_{X=1} X_i = 1$$

Makes it impossible to use the normal mix ratios such as 1:3, 1:5, etc, at a given water/cement ratio. Hence a transformation of the actual components (ingredient proportions) to meet the above criterion is unavoidable. Such transformed ratios

say X₁⁽ⁱ⁾, X₂⁽ⁱ⁾, and X₃⁽ⁱ⁾ for the ith experimental points are called pseudo components. Since X₁, X₂ and X₃ are subject to ∑ X_i = 1, the transformation of cement:RHA:laterite:water at say 0.45 water/cement ratio cannot easily be computed because X₁, X₂, X₃, and X₄ are in pseudo expressions X₁⁽ⁱ⁾, X₂⁽ⁱ⁾, X₃⁽ⁱ⁾ and X₄⁽ⁱ⁾. For the ith experimental point, the transformation computations are to be done.

The arbitrary vertices chosen on the triangle are A₁(0.6:0.4:8:0.45), A₂(0.66:0.34:8.5:0.4) A₃(0.7:0.3:8.1:0.48) and A₄(0.55:0.45:7.8:0.50), based on experience and earlier research reports.



Fig 1 Tetrahedral Simplex

3.2.4 Transformation Matrix

If Z denotes the actual matrix of the ith experimental points, observing from Table 2 (points 1 to 3),

$$BZ = X = 1 \quad \dots \quad (16)$$

where B is the transformed

matrix.

$$\text{Therefore, } B = I \cdot Z^{-1}$$

$$\text{Or } B = Z^{-1} \quad \dots \quad (17)$$

For instance, for the chosen ratios A₁, A₂, A₃ and A₄ (fig 1),

$$\begin{bmatrix} 1.00 & 0.50 & 14.82 & 0.05 \end{bmatrix}$$

$$Z = \begin{pmatrix} 1.00 & 0.51 & 14.58 & 0.52 \\ 1.00 & 0.48 & 15.55 & 0.45 \\ 1.00 & 0.45 & 14.60 & 0.55 \end{pmatrix} \quad (18)$$

From Eqn (17),

$$B = Z^{-1}$$

$$Z^{-1} = \begin{pmatrix} 514.44 & 365.11 & 142.19 & -6.15 \\ -130.90 & 109.44 & 32.62 & -11.16 \\ -21.89 & 15.02 & 6.44 & 0.43 \\ -247.21 & 175.54 & 60.94 & 10.73 \end{pmatrix}$$

Hence,

$$BZ^{-1} = Z \cdot Z^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, for actual component Z, the pseudo component X is given by

$$X \begin{pmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{pmatrix} = B \begin{pmatrix} 514.44 & 365.11 & 142.19 & -6.15 \\ -130.90 & 109.44 & 32.62 & -11.16 \\ -21.89 & 15.02 & 6.44 & 0.43 \\ -39.13 & 8.15 & 7.07 & 23.91 \end{pmatrix} Z \begin{pmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \\ Z_4^{(i)} \end{pmatrix}$$

Table 2 Values for Experiment

N	X ₁	X ₂	X ₃	X ₄	RESPONSE	Z ₁	Z ₂	Z ₃	Z ₄
1	1	0	0	0	Y ₁	1.00	0.50	14.82	0.50
2	0	1	0	0	Y ₂	1.00	0.51	14.58	0.52
3	0	0	1	0	Y ₃	1.00	0.48	15.55	0.45
4	0	0	0	1	Y ₄	1.00	0.45	14.60	0.55
5	1/2	1/2	0	0	Y ₁₂	1.00	0.51	14.70	0.51
6	1/2	0	1/2	0	Y ₁₃	1.00	0.49	15.19	0.48
7	1/2	0	0	1/2	Y ₁₄	1.00	0.48	14.71	0.53
8	0	1/2	1/2	0	Y ₂₃	1.00	0.50	15.07	0.49
9	0	1/2	0	1/2	Y ₂₄	1.00	0.48	14.59	0.54

which gives the X_i(i=1,2,3,4) values in Table 2.

The inverse transformation from pseudo component to actual component is expressed as

$$AX = Z \quad (19)$$

where A = inverse

matrix

$$A = ZX^{-1}$$

From Eqn (16), X = BZ, therefore,

$$A = Z \cdot (BZ)^{-1}$$

$$A = Z \cdot Z^{-1} \cdot B^{-1}$$

$$A = IB^{-1}$$

$$= B^{-1} \quad (20)$$

This implies that for any pseudo component X, the actual component is given by

$$Z \begin{pmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \\ Z_4^{(i)} \end{pmatrix} = B \begin{pmatrix} 1.00 & 0.50 & 14.82 & 0.05 \\ 1.00 & 0.51 & 14.58 & 0.52 \\ 1.00 & 0.48 & 15.55 & 0.45 \\ 1.00 & 0.45 & 14.60 & 0.55 \end{pmatrix} X \begin{pmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{pmatrix} \quad (21)$$

Eqn (21) is used to determine the actual components from points 5 to 10, and the control values from points 11 to 13 (Table 2).

10	0	0	1/2	1/2	Y ₃₄	1.00	0.47	15.08	0.50
Control points									
11	0.25	0.25	0.25	0.25	Y ₁₂₃₄	1.00	0.49	14.89	0.51
12	0.5	0.25	0.25	0	Y ₁₁₂₃	1.00	0.50	14.94	0.49
13	0.25	0.5	0	0.25	Y ₁₂₂₄	1.00	0.49	14.65	0.52

3.2.5 Use of Values in Experiment

During the laboratory experiment, the actual components were used to measure out the appropriate proportions of the ingredients: cement, Rice Hush Ash, laterite and water, for mixing the lateritic concrete materials for casting the samples. The values obtained are presented in Tables in section 5.

3.3 Adequacy of Tests

This is carried out by testing the fit of a second degree polynomial [23]. After the coefficients of the regression equation has been derived, the statistical analysis is considered necessary, that is, the equation should be tested for goodness of fit, and the equation and surface values bound into the confidence intervals. In experimentation following simplex-lattice designs there are no degrees of freedom to test the equation for adequacy, so, the experiments are run at additional so-called test points.

The number of control points and their coordinates are conditioned by the problem formulation and experiment nature. Besides, the control points are sought so as to improve the model in case of inadequacy. The accuracy of response prediction is dissimilar at different points of the simplex. The variance of the predicted response, $S_{\hat{Y}}^2$, is obtained from the error accumulation law. To illustrate this by the second degree polynomial for a ternary mixture, the following points are assumed:

X_i can be observed without errors [23],

the replication variance, S_{Y^2} , is similar at all design points, and

response values are the average of n_i and n_{ij} replicate observations at appropriate points of the simplex

Then the variance $S_{\hat{Y}_i}$ and $S_{\hat{Y}_{ij}}$ will be

$$(S_{\hat{Y}^2})_i = S_{Y^2}/n_i \quad . \quad . \quad (22)$$

$$(S_{\hat{Y}^2})_{ij} = S_{Y^2}/n_{ij} \quad .. \quad . \quad (23)$$

In the reduced polynomial,

$$\hat{Y} = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4 \quad . \quad . \quad (24)$$

If we replace coefficients by their expressions in terms of responses,

$$\beta_i = Y_i \text{ and } \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$$

$$\hat{Y} = Y_1 X_1 + Y_2 X_2 + Y_3 X_3 + Y_4 X_4 + (4Y_{12} - 2Y_1 - 2Y_2) X_1 X_2 + (4Y_{13} - 2Y_1 - 2Y_3) X_1 X_3 + (4Y_{14} - 2Y_1 - 2Y_4) X_1 X_4 + (4Y_{23} - 2Y_2 - 2Y_3) X_2 X_3 + (4Y_{24} - 2Y_2 - 2Y_4) X_2 X_4 + (4Y_{34} - 2Y_3 - 2Y_4) X_3 X_4$$

$$= Y_1(X_1 - 2X_1 X_2 - 2X_1 X_3 - 2X_1 X_4) + Y_2(X_2 - 2X_1 X_2 - 2X_2 X_3 - 2X_2 X_4) + Y_3(X_3 - 2X_1 X_3 + 2X_2 X_3 + 2X_3 X_4) + Y_4(X_4 - 2X_1 X_4 + 2X_2 X_4 + 2X_3 X_4) + 4Y_{12} X_1 X_2 + 4Y_{13} X_1 X_3 + 4Y_{14} X_1 X_4 + 4Y_{23} X_2 X_3 + 4Y_{24} X_2 X_4 + 4Y_{34} X_3 X_4 \quad . \quad . \quad .. \quad (25)$$

Using the condition $X_1 + X_2 + X_3 + X_4 = 1$, we transform the coefficients at Y_i

$$X_1 - 2X_1 X_2 - 2X_1 X_3 - 2X_1 X_4 = X_1 - 2X_1(X_2 + X_3 + X_4) = X_1 - 2X_1(1 - X_1) = X_1(2X_1 - 1) \text{ and so on.} \quad . \quad (26)$$

Thus

$$\hat{Y} = X_1(2X_1 - 1)Y_1 + X_2(2X_2 - 1)Y_2 + X_3(2X_3 - 1)Y_3 + X_4(2X_4 - 1)Y_4 + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{14}X_1X_4 + 4Y_{23}X_2X_3 + 4Y_{24}X_2X_4 + 4Y_{34}X_3X_4$$

Introducing the designation

$$a_i = X_i(2X_i - 1) \text{ and } a_{ij} = 4X_iX_j \quad (27)$$

and using Eqns (22) and (23) give the expression for the variance $S_{\hat{Y}}^2$.

$$S_{\hat{Y}}^2 = S_Y^2 \left(\sum_{1 \leq i \leq q} a_{ii}^2/n_i + \sum_{1 \leq i < j \leq q} a_{ij}^2/n_{ij} \right) \quad (28)$$

If the number of replicate observations at all the points of the design are equal, i.e. $n_i = n_{ij} = n$, then all the relations for $S_{\hat{Y}}^2$ will take the form

$$S_{\hat{Y}}^2 = S_Y^2 \xi / n \quad (29)$$

where, for the second degree polynomial,

$$\xi = \sum_{1 \leq i \leq q} a_i^2 + \sum_{1 \leq i < j \leq q} a_{ij}^2 \quad (30)$$

As in Eqn (30), ξ is only dependent on the mixture composition. Given the replication Variance and the number of parallel observations n , the error for the predicted values of the response is readily calculated at any point of the composition-property diagram using an appropriate value of ξ taken from the curve.

Adequacy is tested at each control point, for which purpose the statistic is built:

$$t = \Delta_Y / (S_{\hat{Y}}^2 + S_Y^2) = \Delta_Y n^{1/2} / (S_Y(1 + \xi))^{1/2} \quad (31)$$

$$\text{where } \Delta_Y = Y_{\text{exp}} - Y_{\text{theory}} \quad (32)$$

and n = number of parallel observations at every point.

The t-statistic has the student distribution, and it is compared with the tabulated value of $t_{\alpha/L}(V)$ at a level of significance α , where L = the number of control points, and V = the

number for the degrees of freedom for the replication variance.

The null hypothesis is that the equation is adequate is accepted if $t_{\text{cal}} < t_{\text{Table}}$ for all the control points.

The confidence interval for the response value is

$$\hat{Y} - \Delta \leq Y \leq \hat{Y} + \Delta \quad (33)$$

$$\Delta = t_{\alpha/L,k} S_{\hat{Y}} \quad (34)$$

where k is the number of polynomial coefficients determined.

Using Eqn (29) in Eqn (34)

$$\Delta = t_{\alpha/L,k} S_Y (\xi/n)^{1/2} \quad (35)$$

4 METHODOLOGY

4.1 Introduction

To be a good structural material, the material should be homogeneous and isotropic. The Portland cement, laterite or concrete are none of these, nevertheless they are popular construction materials [13]. The necessary materials used in the manufacture of the sandcrete block in the study are cement, RHA, laterite and water.

4.2 Materials

The disturbed samples of laterite material were collected at Emene Enugu at the depth of 1.5m below the surface.

The water for use is pure drinking water which is free from any contamination i.e. nill Chloride content, pH =6.9, and Dissolved Solids < 2000ppm. Ordinary Portland cement is the hydraulic binder used in this project and sourced from the Dangote Cement Factory, and assumed to comply with the Standard Institute of Nigeria (NIS) (1974), and kept in an air-tight bag. All samples of the laterite material have properties which conformed to [24]. The RH was collected from the rice husk waste dump at Olo Rice Mill, Ezeagu, Enugu State, and burnt to obtain the ash.

4.3 Preparation of Samples

The sourced materials for the experiment were transferred to the laboratory where they were

allowed to dry. Samples of the laterite were prepared and tested to obtain the moisture content for use in proportioning the components of the sandcrete mix to be prepared. The laterite was sieved to remove debris and coarse particles. The component materials were mixed at ambient temperature. The materials were mixed by weight according to the specified proportions of the actual components generated in Table 1. In all, two hollow blocks of 450mm x225 x150mm for each of the ten experimental points and three control points were cast for the compressive strength test, cured for 28 days after setting and hardening.

4.4 Strength Test

After 28 days of curing, the cubes and blocks were crushed, with dimensions measured before and at the point of shearing, to determine the sandcrete block strength, using the compressive testing machine to the requirements of [25].

5 RESULT AND ANALYSIS

Table 3 Result of the Replication Variance of the Compressive Strength Response for 450x225x150 mm Block

Experiment No (n)	Repetition	Response Y (N/mm ²)	Response Symbol	$\sum Y_r$	\bar{Y}_r	$\sum (Y_r - \bar{Y}_r)^2$	S_r^2
1	1A	0.92	Y ₁	2.01	1.01	0.01	0.00
	1B	1.09					
2	2A	1.44	Y ₂	2.24	1.12	0.20	0.02
	2B	0.80					
3	3A	0.76	Y ₃	1.86	0.93	0.06	0.00
	3B	1.10					
4	4A	1.34	Y ₄	2.34	1.17	0.06	0.00
	4B	1.00					
5	5A	1.02	Y ₁₂	1.79	0.90	0.03	0.00
	5B	0.77					
6	6A	2.21	Y ₁₃	3.15	1.58	0.81	0.07
	6B	0.94					
7	7A	1.05	Y ₁₄	1.94	0.97	0.01	0.00
	7B	0.89					
8	8A	0.87	Y ₂₃	1.92	0.96	0.02	0.00
	8B	1.05					
9	9A	1.08	Y ₂₄	2.58	1.29	0.09	0.01
	9B						

5.1 Determination of Replication Error and Variance of Response

To raise the experimental design equation models by the lattice theory approach, two replicate experimental observations were conducted for each of the design and control points.

Hence we have below, the table of the results (Table 3) which contain the results of two repetitions each of the 10 design points plus the three Control Points of the (4,2) simplex lattice, and show the mean and variance values per test of the observed response, using the following mean and variance equations below:

$$\bar{Y} = \frac{\sum(Y_r)}{r} \quad \dots \quad (36)$$

where \bar{Y} is the mean of the response values and $r=1,2$.

$$S_r^2 = \frac{\sum[(Y_i - \bar{Y}_i)^2]}{(n-1)} \quad \dots \quad (37)$$

where n = 13.

		1.50					
10	10A 10B	1.28 1.02	Y ₃₄	2.30	1.15	0.03	0.00
Control Points							
11	11A 11B	0.98 0.99	C ₁	1.97	0.99	0.00	0.00
12	11A 11B	1.11 1.01	C ₂	2.12	1.06	0.01	0.00
13	13A 13B	2.01 0.77	C ₃	2.78	1.39	0.77	0.06

Σ2.10

Replication Variance

$$S_{Yc}^2 = (\sum S_i^2)/(n-1) = 2.10/12 = 0.17$$

Replication Error

$$S_{Yc} = (S_{Yc}^2)^{1/2} = 0.17^{1/2} = 0.42$$

5.2 Determination of Regression Equation for the Compressive Strength

From Eqns 15 and Table 3 the coefficients of the reduced second degree polynomial is determined as follows:

$$\beta_1 = 1.01$$

$$\beta_2 = 1.12$$

$$\beta_3 = 0.93$$

$$\beta_4 = 1.17$$

$$\beta_{12} = 4(0.90) - 2(1.01) - 2(1.12) = -0.67$$

$$\beta_{13} = 4(1.58) - 2(1.01) - 2(0.93) = 2.43$$

$$\beta_{14} = 4(0.97) - 2(1.01) - 2(1.17) = -0.47$$

$$\beta_{23} = 4(0.90) - 2(1.12) - 2(0.93) = -0.26$$

$$\beta_{24} = 4(1.29) - 2(1.12) - 2(0.93) = 0.58$$

$$\beta_{34} = 4(1.15) - 2(0.93) - 2(1.17) = 0.40$$

Thus, from Eqn (11),

$$\hat{Y} = 1.01X_1 + 1.12X_2 + 0.93X_3 + 1.17X_4 - 0.67X_1X_2 + 2.43X_1X_3 - 0.47X_1X_4 - 0.26X_2X_3 + 0.58X_2X_4 + 0.40X_3X_4 \dots \quad (38)$$

Eqn (38) is the mathematical model of the compressive strength of the hollow sandcrete block based on the 28-day strength.

5.3 Test of Adequacy of the Compressive Strength Model

Eqn 38, the equation model, will be tested for adequacy against the controlled experimental results.

We recall our statistical hypothesis as follows:

1. Null Hypothesis (H₀): There is no significant difference between the experimental values and the theoretical expected results of the compressive strength.
2. Alternative Hypothesis (H₁): There is a significant difference between the experimental values and the theoretical expected results of the compressive strength.

5.4 T-test for the Compressive Strength Model

If we substitute for X_i in Eqn 38 from Table 2, the theoretical predictions of the response (\hat{Y}) can be obtained. These values can be compared with the

experimental results (Table 2). For the t-test (Table 3), a , ξ , t and Δ_y are evaluated using Eqns (27), (30), (31) and (32) respectively.

Table 4 t-Test for the Test Control Points

N	CN	I	J	a_i	a_{ij}	a_i^2	a_{ij}^2	ξ	\hat{Y}	\hat{Y}_a	Δ_y	T
1	C ₁	1	2	-0.125	0.250	0.016	0.063	0.469	0.99	1.18	0.51	0.86
		1	3	-0.125	0.250	0.016	0.063					
		1	4	-0.125	0.250	0.016	0.063					
	2	3	-0.125	0.250	0.016	0.063						
	2	4	-0.125	0.250	0.016	0.063						
	3	4	-0.125	0.250	0.016	0.063						
					0.094	0.375						
2	C ₂	1	2	0.000	0.500	0.000	0.250	0.563	1.06	1.22	0.47	1.28
		1	3	0.000	0.500	0.000	0.250					
		1	4	0.000	0.000	0.000	0.000					
	2	3	0.000	0.250	0.000	0.063						
	2	4	0.000	0.000	0.000	0.000						
	3	4	0.000	0.000	0.000	0.000						
					0.000	0.563						
3	C ₃	1	2	-0.125	0.500	0.016	0.250	0.656	1.39	1.06	0.11.	0.28
		1	3	-0.125	0.000	0.016	0.000					
		1	4	-0.125	0.250	0.016	0.063					
		2	3	-0.125	0.000	0.016	0.000					
		2	4	-0.125	0.500	0.016	0.250					
		3	4	-0.125	0.000	0.016	0.000					
						0.094	0.563					

Significance level $\alpha = 0.05$,

$$\text{i.e. } t_{\alpha/L}(V_c) = t_{0.05/3}(13), \quad \text{where}$$

L = number of control point.

From the Student-T Table, the tabulated value of $t_{0.05/3}(13)$ is found to be 3.01 which is greater than any of the calculated t-values in Table 4. Hence we can accept the Null Hypothesis.

From Eqn (35), with $k=3$ and $t_{\alpha/k,v} = t_{0.05/k}(13) = 3.01$,
 $\Delta = 1.53$ for C_{1234} , 1.57 for C_{1124}
 $= 0.26$, and 1.62 for C_{1224} ,
which satisfies the confidence interval equation of Eqn (33) when viewed against the response values in Table 3.

5.5 Computer Program

The computer program is developed for the model. In the program any desired Compressive Strength can be specified as an input and the computer processes and prints out possible combinations of mixes that match the property, to the following tolerance:

Compressive Strength -
0.001 N/mm²,

Interestingly, should there be no matching combination, the computer informs the user of this. It also checks the maximum value obtainable with the model.

5.6 Choosing a Combination

It can be observed that the strength of 1.40 N/sq mm yielded 5 combinations (APPENDIX A). To accept any particular proportions depends on the factors such as workability and cost of the resultant sandcrete.

6 CONCLUSION AND COMMENDATION

6.1 Conclusion

Henry Scheffe's simplex design was applied successfully to prove that the compressive strength of sandcrete block is a function of the proportion of the ingredients (cement, RHA, laterite and water), but not the quantities of the materials.

The maximum compressive strength obtainable with the compressive strength model is 1.57 N/sq mm. See the computer run outs (APPENDIX A) which show all the possible sandcrete mix options for the desired compressive strength property, and the attainable compressive strength for the chosen ratio. The choice of any of the mixes is the user's.

One can also draw the conclusion that the maximum values achievable, within the limits of experimental errors, is quite below that obtainable using sand as aggregate. This is due to the weaker binding strength of the RHA component.

It can be observed that the task of selecting a particular mix proportion out of many options is not easy, if workability and other demands of the resulting sandcrete has to be satisfied. This is an important area for further research work.

The project work is a great advancement in the estimation of the usefulness of RHA in sandcrete mortar production in regions where cement may be too costly to buy with the ubiquity of rice husk.

6.2 Recommendations

From the foregoing study, the following could be recommended:

- i) The model can be used for the optimization of the strength of sandcrete made from cement, RHA, laterite and water.
- ii) Rice Hush Ash binder cannot adequately substitute cement for heavy construction.
- iii) More research work need to be done in order to match the computer recommended mixes with the workability of the resulting sandcrete.
- iii) The accuracy of the model can be improved by taking higher order polynomials of the simplex.

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APPENDIX A

'QBASIC BASIC PROGRAM THAT OPTIMIZES THE PROPORTIONS OF SANDCRETE MIXES USING THE SCHEFFE'S MODEL FOR CONCRETE COMPRESSIVE STRENGTH

CLS

C1\$ = "(ONUAMAH.HP) RESULT OUTPUT "

C2\$ = "A COMPUTER PROGRAM "

C3\$ = "ON THE OPTIMIZATION OF A 4-COMPONENT SANDCRETE MIX"

PRINT C2\$ + C1\$ + C3\$

PRINT

'VARIABLES USED ARE

'X1, X2, X3, X4, Z1, Z2, Z3, Z4, Z\$, YT, YTMAX, DS

INPUT "ARE MIX RATIOS KNOWN AND THE ATTAINABLE STRENGTH NEEDED?, CHOOSE Y= YES OR N=NO"; OPTION\$

IF OPTION\$ = "Y" THEN

INPUT "X1="; X1

INPUT "X2="; X2

INPUT "X3="; X3

INPUT "X4="; X4

X = X1 + X2 + X3 + X4

X1 = X1 / X; X2 = X2 / X; X3 = X3 / X; X4 = X4 / X

GOTO 25

ELSE

GOTO 20

END IF

INITIALISE I AND YTMAX

```

20 I = 0: YTMAX = 0

FOR MX1 = 0 TO 1 STEP .01
FOR MX2 = 0 TO 1 - MX1 STEP .01
FOR MX3 = 0 TO 1 - MX1 - MX2 STEP .01
MX4 = 1 - MX1 - MX2 - MX3
YTM = 1.01 * MX1 + 1.12 * MX2 + 0.93 * MX3
+ 1.17 * MX4 - 0.67 * MX1 * MX2 + 2.43 * MX1 *
MX3 - 0.47 * MX1 * MX4 + -0.26 * MX2 * MX3 + 0.58
* MX2 * MX4 + 0.40 * MX3 * MX4
IF YTM >= YTMAX THEN YTMAX = YTM
NEXT MX3
NEXT MX2
NEXT MX1
INPUT "ENTER DESIRED STRENGTH, DS = ";
DS

'PRINT OUTPUT HEADING
PRINT
25 PRINT TAB(1); "No"; TAB(10); "X1"; TAB(18);
"X2"; TAB(26); "X3"; TAB(34); "X4"; TAB(40);
"YTHEORY"; TAB(50); "Z1"; TAB(58); "Z2";
TAB(66); "Z3"; TAB(74); "Z4"
IF OPTION$ = "Y" THEN 30
PRINT
'COMPUTE THEORETICAL STRENGTH, YT
FOR X1 = 0 TO 1 STEP .01
FOR X2 = 0 TO 1 - X1 STEP .01
FOR X3 = 0 TO 1 - X1 - X2 STEP .01
X4 = 1 - X1 - X2 - X3
30 YT = 1.01 * X1 + 1.12 * X2 + 0.93 * X3 + 1.17 *
X4 - 0.67 * X1 * X2 + 2.43 * X1 * X3 - 0.47 * X1 * X4 +
-0.26 * X2 * X3 + 0.58 * X2 * X4 + 0.40 * X3 * X4

IF OPTION$ = "Y" THEN 40
IF ABS(YT - DS) <= .00005 THEN
'PRINT MIX PROPORTION RESULTS
Z1 = X1 + X2 + X3 + X4: Z2 = .50 * X1 + .515 *
X2 + .48 * X3 + .45 * X4: Z3 = 14.82 * X1 + 14.58 * X2
+ 15.55 * X3 + 14.60 * X4: Z4 = .50 * X1 + .52 * X2 +
.45 * X3 + .55 * X4
40 I = I + 1
PRINT TAB(1); I; USING "###.###"; TAB(7);
X1; TAB(15); X2; TAB(23); X3; TAB(32); X4;
TAB(40); YT; TAB(48); Z1; TAB(56); Z2; TAB(64);
Z3; TAB(72); Z4
PRINT
PRINT
IF OPTION$ = "Y" THEN 540

```

```

IF (X1 = 1) THEN 550
ELSE
IF (X1 < 1) THEN GOTO 150
END IF

150 NEXT X3
NEXT X2
NEXT X1
IF I > 0 THEN 550
PRINT
PRINT "SORRY, THE DESIRED STRENGTH IS
OUT OF RANGE OF MODEL"
GOTO 600
540 PRINT TAB(5); "THE ATTAINABLE
STRENGTH IS "; YT; "N/mm2"
GOTO 600
550 PRINT TAB(5); "THE MAXIMUM VALUE
PREDICTABLE BY THE MODEL IS "; YTMAX; "N
/Sq mm "
600 END

```

A COMPUTER PROGRAM (ONUAMAH.HP)
RESULT OUTPUT ON THE OPTIMIZATION OF
A 4-COMPONE
NT SANDCRETE MIX

ENTER DESIRED STRENGTH, DS = ? 1.4

No	X1	X2	X3	X4	YTHEORY	Z1	Z2
1	0.320	0.020	0.520	0.140	1.400	1.000	0.483
	15.164	0.481					
2	0.320	0.080	0.580	0.020	1.400	1.000	0.489
	15.220	0.474					
3	0.340	0.040	0.500	0.120	1.400	1.000	0.485
	15.149	0.482					
4	0.480	0.010	0.350	0.160	1.400	1.000	0.485
	15.038	0.491					

5 0.500 0.080 0.360 0.060 1.400 1.000 0.491
15.050 0.487

No	X1	X2	X3	X4	YTHEORY	Z1	Z2
	Z3	Z4					

THE MAXIMUM VALUE PREDICTABLE BY
THE MODEL IS 1.578128 N / Sq mm

1	0.059	0.030	0.881	0.030	1.077	0.000	0.000
	0.000	0.000					

THE ATTAINABLE STRENGTH IS 1.076996
N/mm2

A COMPUTER PROGRAM (ONUAMAH.HP)
RESULT OUTPUT ON THE OPTIMIZATION OF
A 4-COMPONE
NT SANDCRETE MIX

Press any key to continue

ENTER DESIRED STRENGTH, DS = ? 1.6

No	X1	X2	X3	X4	YTHEORY	Z1	Z2
	Z3	Z4					

SORRY, THE DESIRED STRENGTH IS OUT OF
RANGE OF MODEL

Press any key to continue

A COMPUTER PROGRAM (ONUAMAH.HP)
RESULT OUTPUT ON THE OPTIMIZATION OF
A 4-COMPONE
NT SANDCRETE MIX

ARE MIX RATIOS KNOWN AND THE
ATTAINABLE STRENGTH NEEDED?,CHOOSE
Y= YES OR N=NO?

Y

X1=? 1

X2=? .5

X3=? 14.82

X4=? .5